Introduction and Motivation to Measurement Uncertainty Concepts

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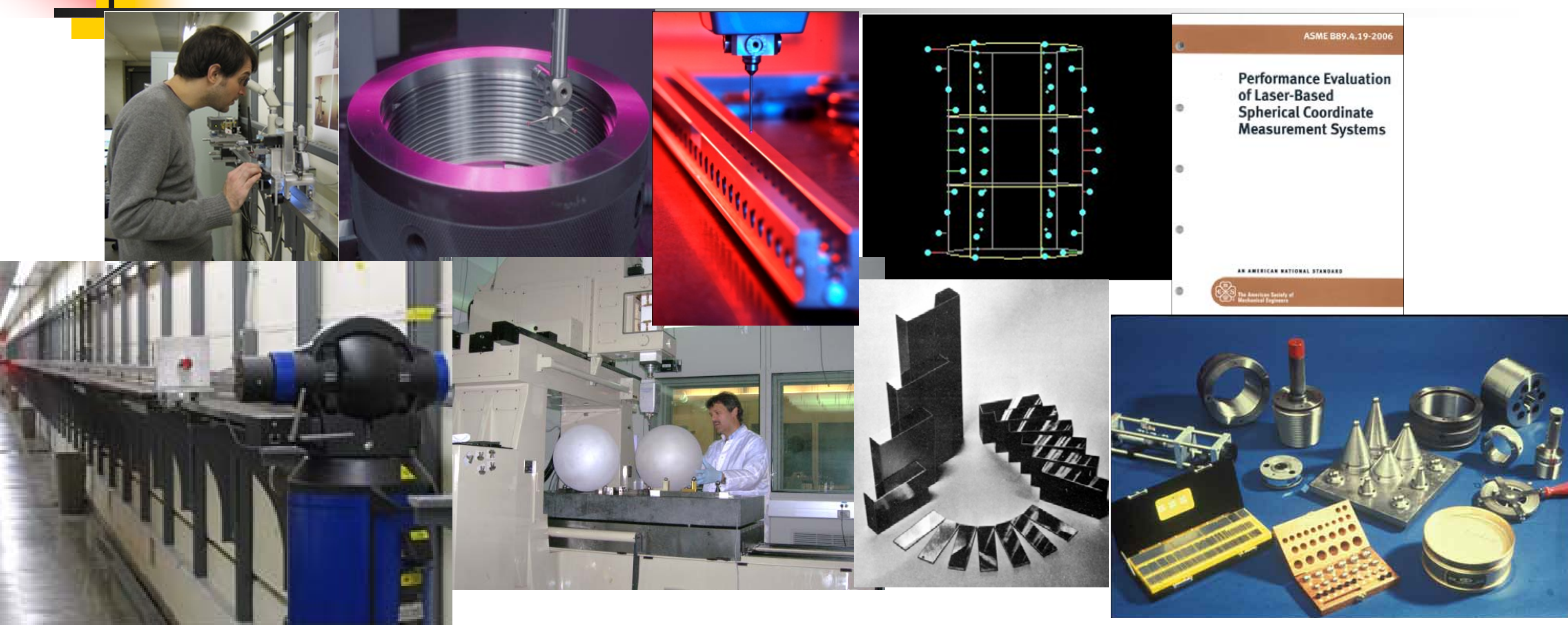
Providing high accuracy measurements with rigorous metrological traceability since 1901



Workshop Outline

- Introduction & Economic Issues
- Basic Terminology & Uncertainty Concepts
- ASME B89.7 Documentary Standards
- Measurement Uncertainty Evaluation

Motivation & Economics of Measurement Uncertainty

- Advanced economies, like the US, produce high value products $\Rightarrow$ dimensional control of features (GD&T) $\Rightarrow$ measurement uncertainty must be known and significantly less than the tolerance.

- To understand or control a maufacturing process, measurements with known uncertainty are needed.

- Uncertain measurements led to uncertain decisions and increased costs

Decisions Generally Have Economic Consequences

- Measurements :
  - Provide Information to Make Better Decisions
  - Cost Money and Time
- Manage the Risk of Making Decisions Through Uncertainty Evaluation
  - The risks can be managed by measurement uncertainty analysis – a technical issue
  - A decision is based on cost analysis, and hence is a matter of business economics
  - Bad decisions cost money
  - An economically optimal decision balances costs vs uncertainty

General Concepts of Uncertainty

- What is Measurement Uncertainty?

  Uncertainty of a measurement means doubt about the validity of the result of the measurement (GUM 2.2.1)

- Measurement uncertainty represents our *state of knowledge* regarding a measurement result.
  - When new information is available our knowledge changes and the uncertainty statement needs to be updated to reflect this new information.

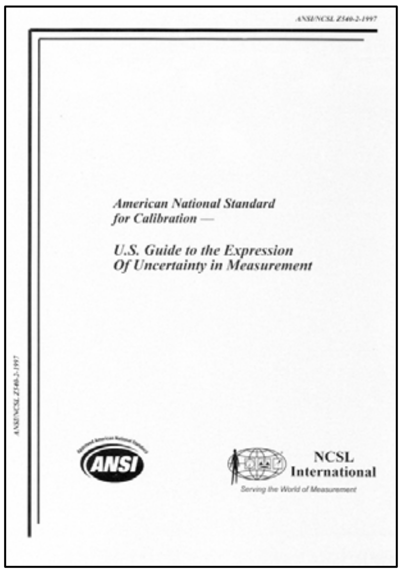
Benefits of Uncertainty Evaluation

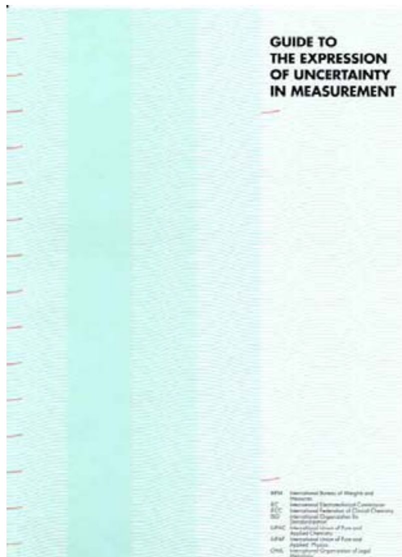
- Measurement Traceability
  - Required for calibration labs per ISO 17025
- Select right tool for the job
  - e.g. meeting 4:1 Tolerance to Uncertainty ratio
- Economics of workpiece accept/reject decisions
  - Optimizing decision rules
- Effective use of $ for improved accuracy
  - Identify and address the largest uncertainty sources

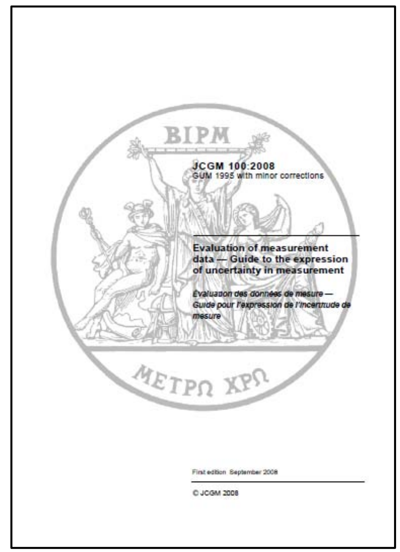
Uncertainty Standardization

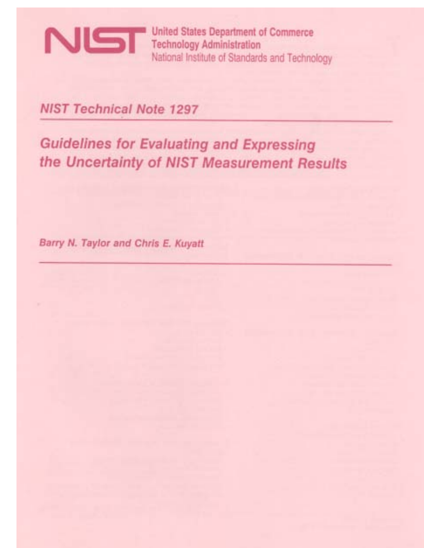
- The formulation of Uncertainty presented today is based on the *Guide to the Expression of Uncertainty in Measurement*, the "GUM"
- This provides a internationally accepted and well defined methodology for all measurements
  - Used by all NMIs, e.g. NIST, PTB, NPL...
  - Used by all ISO 17235 accredited calibration labs, e.g. A2LA, LAB, NVLAP, accredited.
  - Used in all modern ASME & ISO standards
  - Is a US National Standard (NCSL Z540-2)

The GUM









ANSI/NCSL Z540-2-1997 (same as GUM) is an American National Standard
http://www.bipm.org/en/publications/guides/gum.html (free download)
http://physics.nist.gov/Pubs/guidelines/TN1297/tn1297s.pdf (free download)

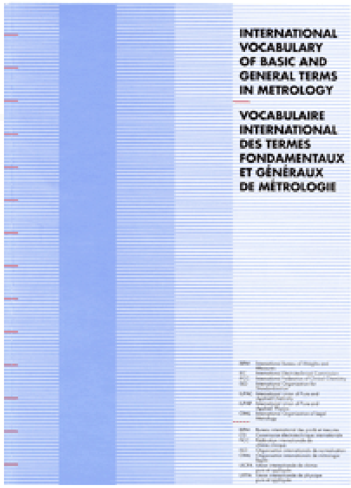
Mistakes are NOT Uncertainty Sources

Mistakes and Blunders associated with taking, recording, or analyzing measurement data and uncertainty are **NOT** considered an uncertainty source for purposes of its calculation (3.4.7)

Example: Do Not include an uncertainty source for data transcription errors, but do review your data carefully!

"Random results are the consequences of random procedures"

Basic Terminology: The VIM



www.bipm.org/en/publications/guides/vim.html

(ISO Guide 99)

Beware of Jabberwocky:

"The big red sound smelled like sandpaper."

"A direct uncertainty of – 5 μm resulted from a type B error associated with the systematic uncertainty of the resolution of the true value to yield a 95% confidence interval."

Basic Terminology: Speak like a Pro!

- Calibration & Inspection
- Measurand
- Conformance / Non-Conformance
- Accuracy
- Uncertainty
- Error
- Bias (Systematic Error)

Calibration & Inspection

For purposes of this workshop, we define:

- Calibration: The measurement process of assigning a value and its (GUM) uncertainty to an artifact, workpiece or instrument result in a documented manner; e.g. the length of a gauge block and the associated length uncertainty in a calibration report
- Inspection: The measurement and decision process associated with the acceptance or rejection of an artifact, workpiece or instrument

The Measurand

- The particular quantity subject to measurement (GUM B.2.9)
- A set of specifications (instructions) (GUM D.1)
- NOT a number or value
- Specifies the values, i.e. the “conditions”, of all the potential influence quantities so that (ideally) ONE “true value” can be realized.

The Measurand & True Values

- Failure to adequately define the measurand <u>IS</u> a source of measurement uncertainty (GUM 3.3.2) and can result in Multiple True Values!!! (GUM B.2.3)

- A True Value is the result of a perfect instrument measuring an infinite number of points on the surface while fully complying with the definition of the measurand

- GD&T reduces the number of true values...
  - But not reduced to zero...
  - We don't follow the instructions of the measurand...

The Realized Quantity of a Measurement

- Typically, the value realized by the measurement system is not consistent with the definition of the measurand and a CORRECTION must be applied to yield a result that does satisfy the measurand. Example, length measurement at 22 °C must be corrected to 20 °C
- The uncertainty in the correction is an uncertainty contributor
- We frequently choose, or are forced, to realize a quantity other than the measuand and thus are required to make corrections.

Conformance / Non-Conformance

- Conforming:
  - Having at least one true value lying within or on the boundary of a stated tolerance interval.
- Non-Conforming:
  - Having all true values lying outside the boundary of a stated tolerance interval.
- Note:
  - Metrologists do NOT know the true value, and hence do not know if a workpiece is conforming... we can only decide acceptance or rejection.

Accuracy

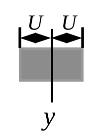
- “The closeness of the agreement between the result of a measurement and a true value of the measurand.” (GUM B.2.14)
  - the measurand is the particular quantity subject to measurement
  - the true value is unknowable
- Accuracy is a qualitative concept
- Uncertainty is the quantitative statement of accuracy
- “Precision” is not accuracy
  - (I prefer not to use the word “precision” as a quantitative metrological quantity)

Measurement Uncertainty *U*

- (Expanded) Uncertainty of a measurement
  - "A parameter [number] associated with the result of a measurement, that characterizes the dispersion [spread] of the values that could reasonably be attributed to the measurand" (GUM B.2.18)
- The issue of <u>reasonable values</u> is fundamental to the GUM; reasonableness allows "expert judgment" in addition to experimental data

Measurement Uncertainty *U* (continued)

- (Expanded) Uncertainty is a <u>positive</u> number, not an interval or probability distribution
  - Incorrect: $U = \pm 5\ \mu m$ Correct: $U = 5\ \mu m$
  - Stating measurement results: $Y = y \pm U$ OR $y - U \leq Y \leq y + U$
  - Note: the uncertainty is centered about the measured value creating an uncertainty interval of width $2U$



    - The VIM calls the “uncertainty interval” the “coverage interval”
- Uncertainty is an attribute of a measurement result
  - Instruments do NOT have uncertainty, measurement results do!

Measurement Uncertainty *U* (continued)

- Default (expanded) uncertainty corresponds to a level of confidence of ≈ 95 %
  - Do not use “confidence level” or “confidence interval”
  - This means you will bet $95 against $5 that the “true value” of the measurand ***Y***, with a best estimated value ***y*** (from measurement), lies in the uncertainty interval:

$$y - U \leq Y \leq y + U$$

- Use uncertainty to describe our ignorance about the “true value” of the measurand

Measurement Error

- "The Measured value minus the True value"

  (GUM B.2.19) $\mathbf{E = \mathit{y} - T}$

- Errors can have either a positive or negative sign

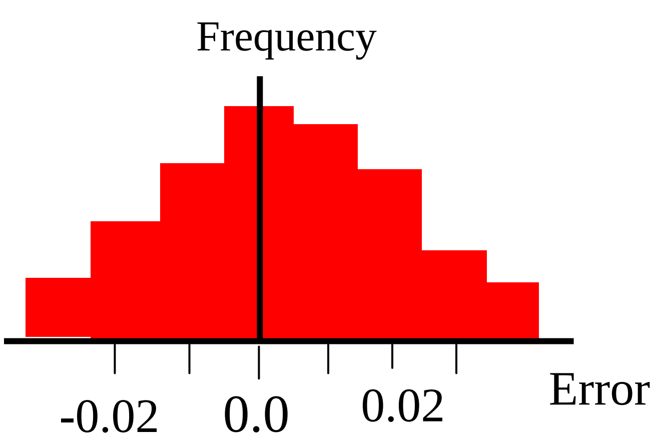
- Since the true value is unknown, the error is unknown -- hence we can only estimate errors – but often we have very good estimates!

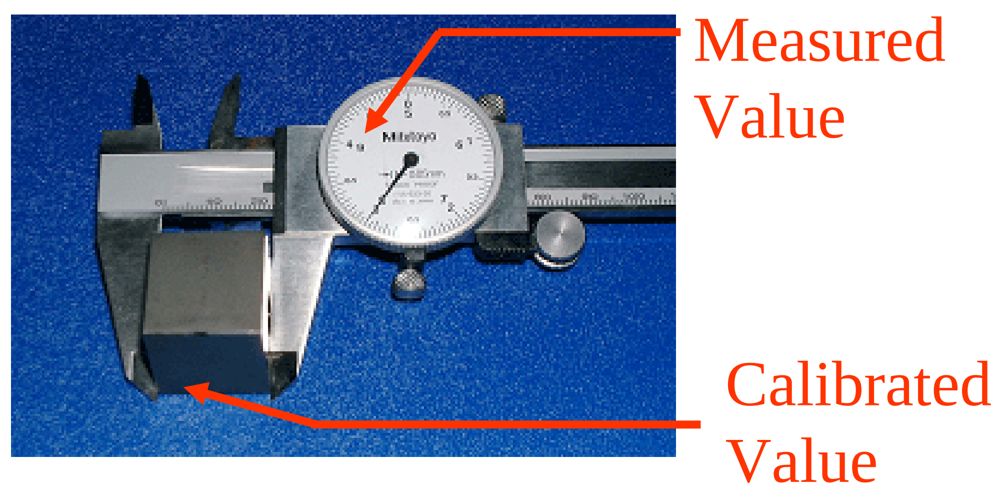
Measurement Error (continued)

- For the measurement of most workpieces, the true value is unknown (which is why we are measuring), hence the error is unknown. In this case we should speak of uncertainty not error
- We can ONLY determine errors when we have an estimate of the true value, e.g. when we measure CALIBRATED objects!
- Typically, the only time we estimate errors is during a calibration.

Measurement Error (continued)

- Two different measurements of a gauge block with a caliper may yield two different measurement results and hence two different errors.
- Repeated measurements will yield a distribution of errors for the measurand; This is telling us something about the measurement uncertainty

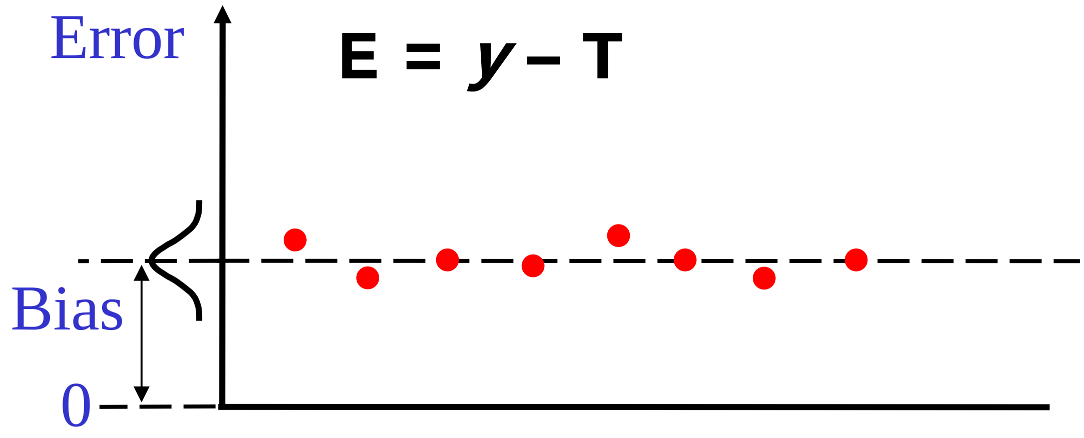




Systematic Measurement Error (Measurement Bias)

- “The component of measurement error that in replicate measurements remains constant or varies in a predictable manner” (VIM 2.17)
- The (mathematical) expectation value of the error and estimated as the arithmetic average error
- The GUM strongly recommends correcting for all significant systematic errors in the measurement

Repeated measurements with good reproducibility but with a large uncorrected bias

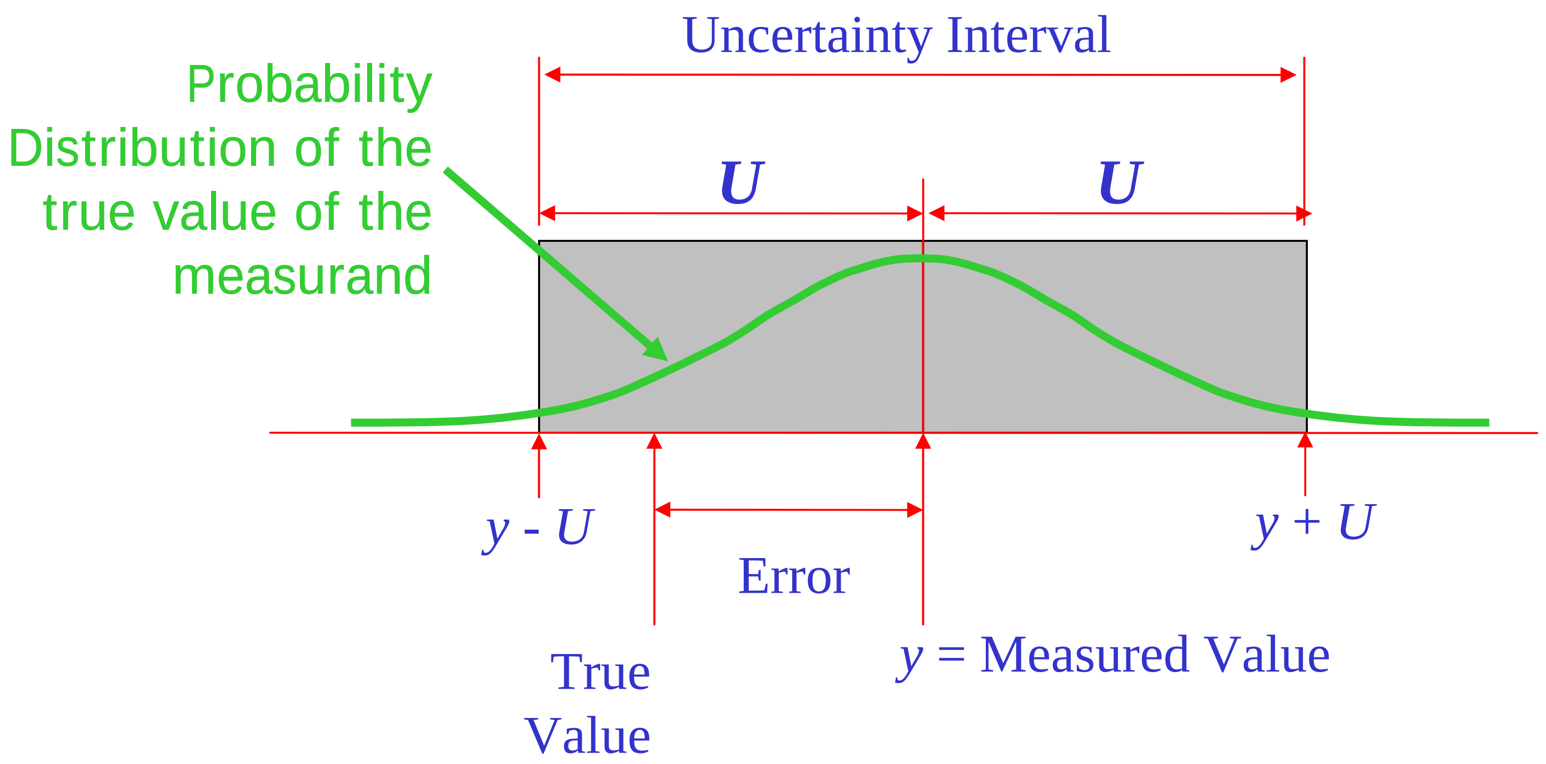


Note: Gage repeatability and reproducibility (GR&R) studies typically use uncalibrated workpieces and hence report only variations not errors. Systematic error (bias) is not observable with uncalibrated workpiece GR&R studies.

Some Sources of Systematic Error

- Uncorrected thermal errors
- Workpiece deflection under probe contact
- Incorrect algorithm fitting
- Fixture induced distortions
- Point sampling strategy
- Systematic errors in instrument
- Incorrect compensations
  - Workpiece temperature, CTE, index of refraction...

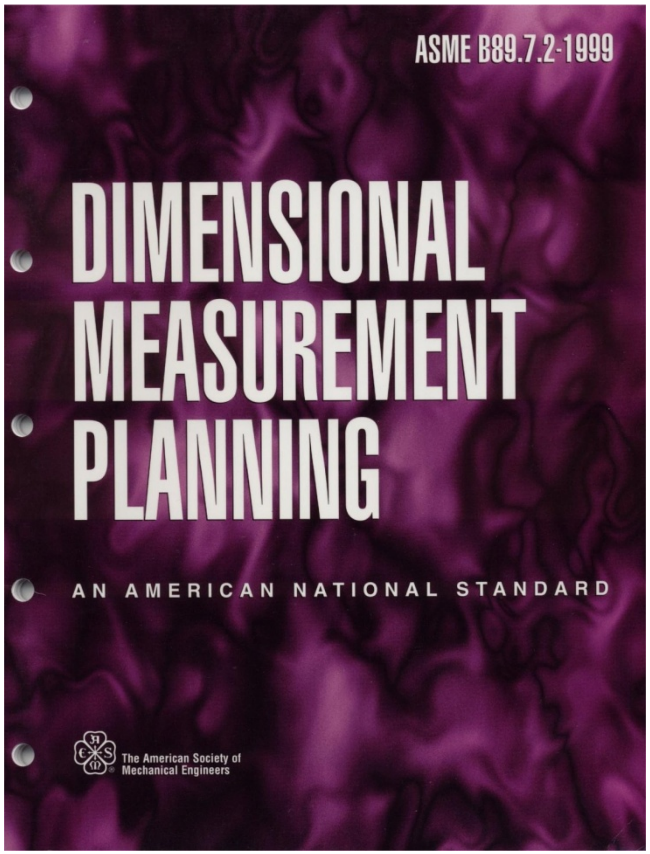
Error, Expanded Uncertainty, Uncertainty Interval, Measured Value, & True Value



US Documentary Uncertainty Standards and Reports
The ASME B89.7 Series…

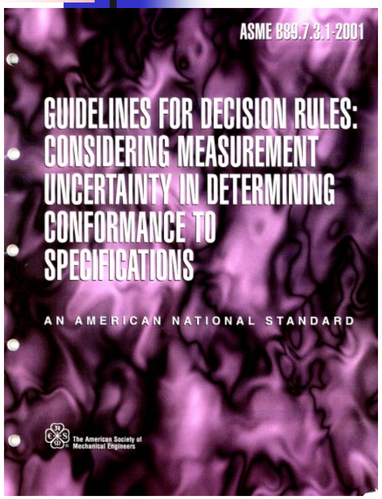
- Addresses the issue of measurement uncertainty in dimensional measurements
  - Particularly industrial measurements
  - Considers the "lifecycle" of uncertainty
    - B89.7.2 Dimensional Measurement Planning
    - B89.7.3.1 Decision rules for accept / reject decisions
    - B89.7.3.2 Simplified GUM evaluation
    - B89.7.3.3 Resolving differing uncertainty evaluations
    - B89.7.4.1 Risk analysis
    - B89.7.5 Measurement traceability
  - www.asme.org

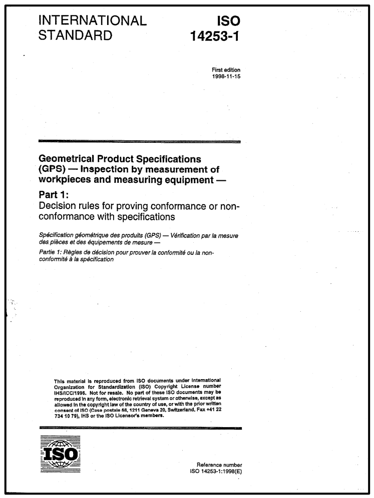
Dimensional Measurement Planning 1999 and 2012(?) [under revision]



- Provides the dimensional measurement planner with overview and check list of requirements
- Discusses rational of measurements
- Reviews other Standards in B89.7 series

Decision Rules used for inspection…
B89.7.3.1 (2001) & ISO 14253-1 (1998)

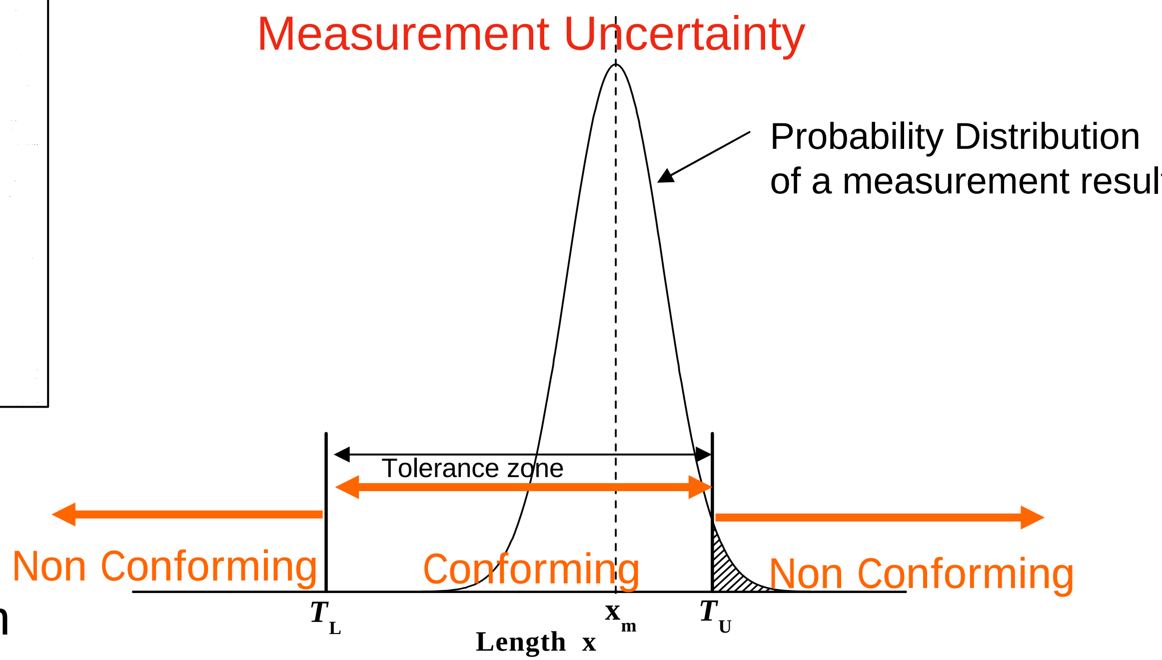



INTERNATIONAL STANDARD

ISO 14253-1

Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment —

Part 1: Decision rules for proving conformance or non-conformance with specifications



Given:

- The product specification
- The measurement result
- The measurement uncertainty

A decision rule determines acceptance/rejection

Note: A workpiece is Conforming to specifications if the true value of the measurand is within the specification zone

Decision Rules

Decision Rule:

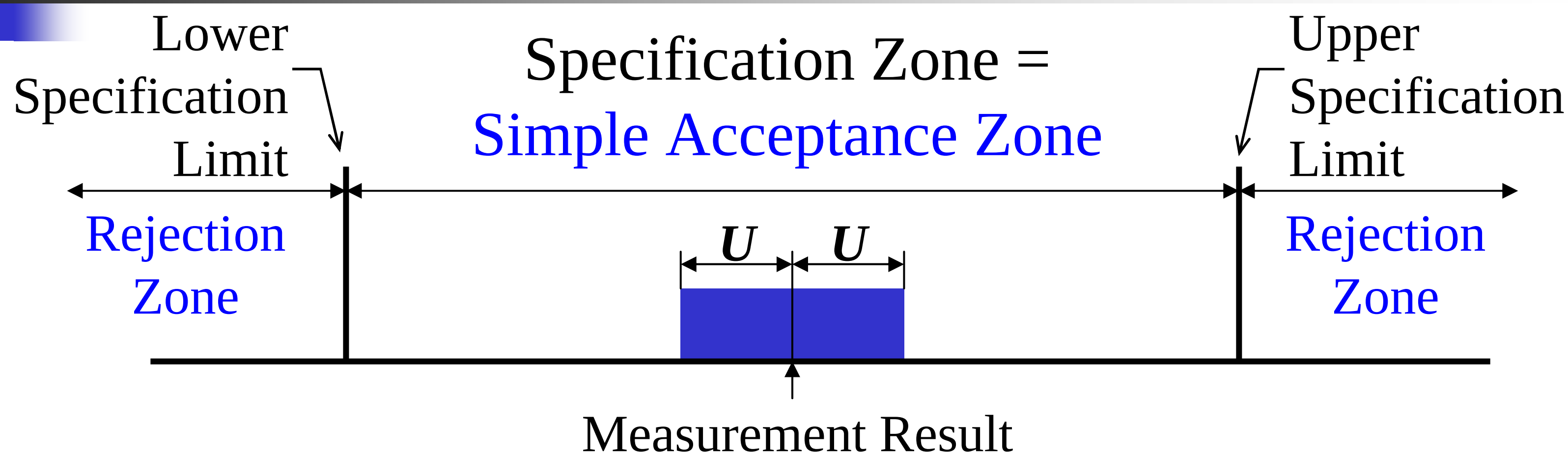
A documented rule, meeting the requirements of section 3 of B89.7.3.1, *that describes how measurement uncertainty will be allocated with regard to accepting or rejecting a product according to its specification and the result of a measurement.*

B89.7.3.1

Section 3, B89.7.3.1, Requirements for Decision Rules

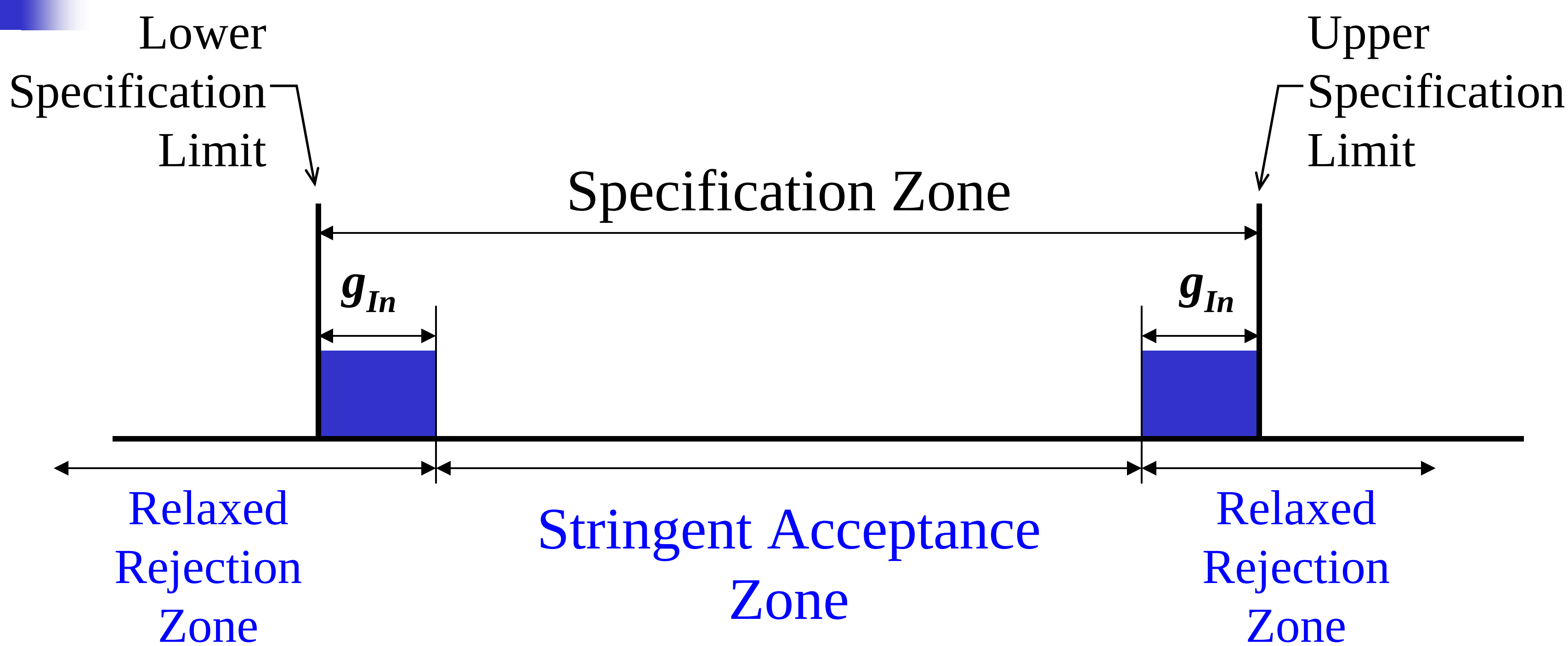
- Zone Identification (all measurement outcomes must be identified)
- Decision Outcome (all measurement results must yield a decision)
- Repeated Measurements Policy
- Data Rejection Policy

Simple 4:1 Acceptance Decision Rule



- The measurement uncertainty interval is of width 2 *U*
- The uncertainty interval is ≤ ¼ the product's specification zone
- Hence the "measurement capability index" $C_m = 4$
- Acceptance if result in specification zone; rejection otherwise
- The measurement value shown results in product acceptance

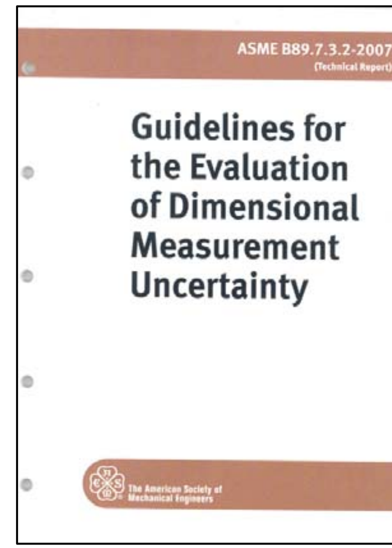
Stringent Acceptance & Relaxed Rejection



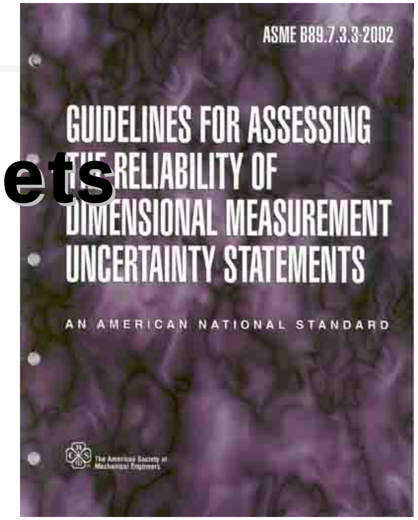
- "g" is known as the guard band, typically expressed as a percentage of *U*
- The default rule of ISO 14253-1 is stringent acceptance with a 100 % *U* guard band.
- B89.7.3.1 treats the guard band as a business decision.

ASME B89.7.3.2
Simplified Uncertainty Evaluation

- Dimensional Measurement Uncertainty Evaluation for Industrial Practitioners
  - Avoids:
    - partial derivatives
      - All input quantities in units of length
    - degrees of freedom
      - Suggestions to minimize this effect
    - correlation
      - Suggestions to avoid correlated quantities
  - Provides: Worked Examples



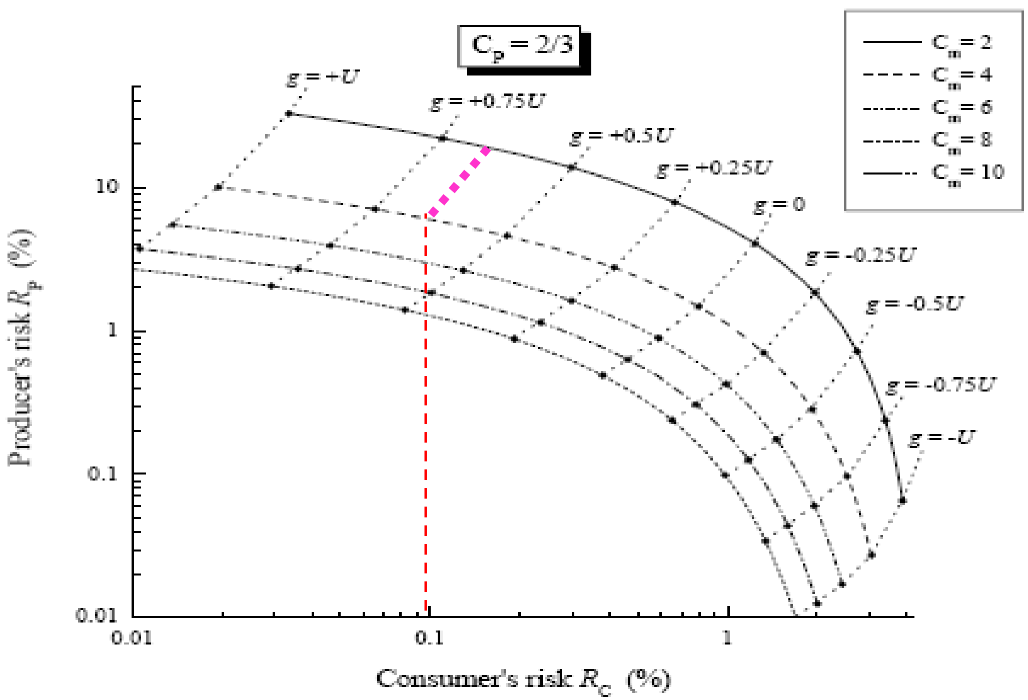
ASME B89.7.3.3: Assessing the Reliability of Uncertainty Statements

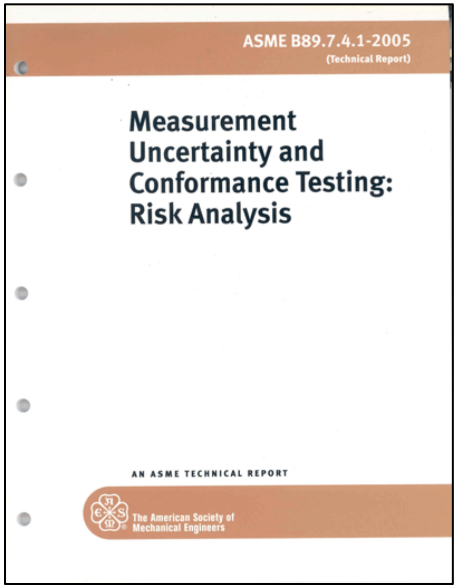


- **Comparison of Uncertainty Budgets**
  - Accounting for Uncertainty Sources
  - Magnitudes of Uncertainty Components
  - Effects of Uncertainty Sources
  - Third Party Review and Accreditation
- **Direct Measurement of the Measurand**
  - Historical Measurements
  - Round Robins
  - Reproducibility Measurements (e.g. GR&R)
  - Measurement of Calibrated Artifacts
  - Third Party Measurements

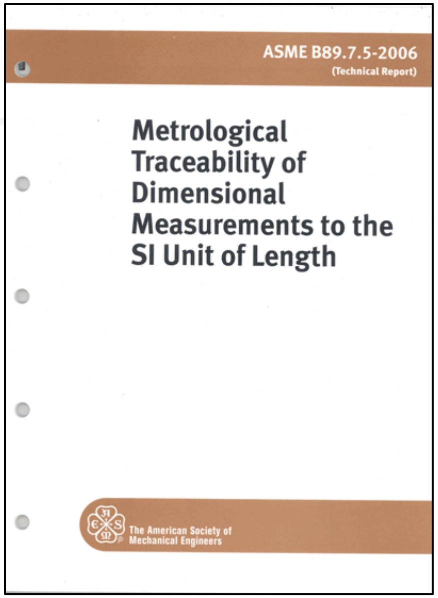
ASME B89.7.4: Measurement Uncertainty and Conformance Testing: Risk Analysis

- ASME B89.7.4 Determines the guardband needed based on business decisions



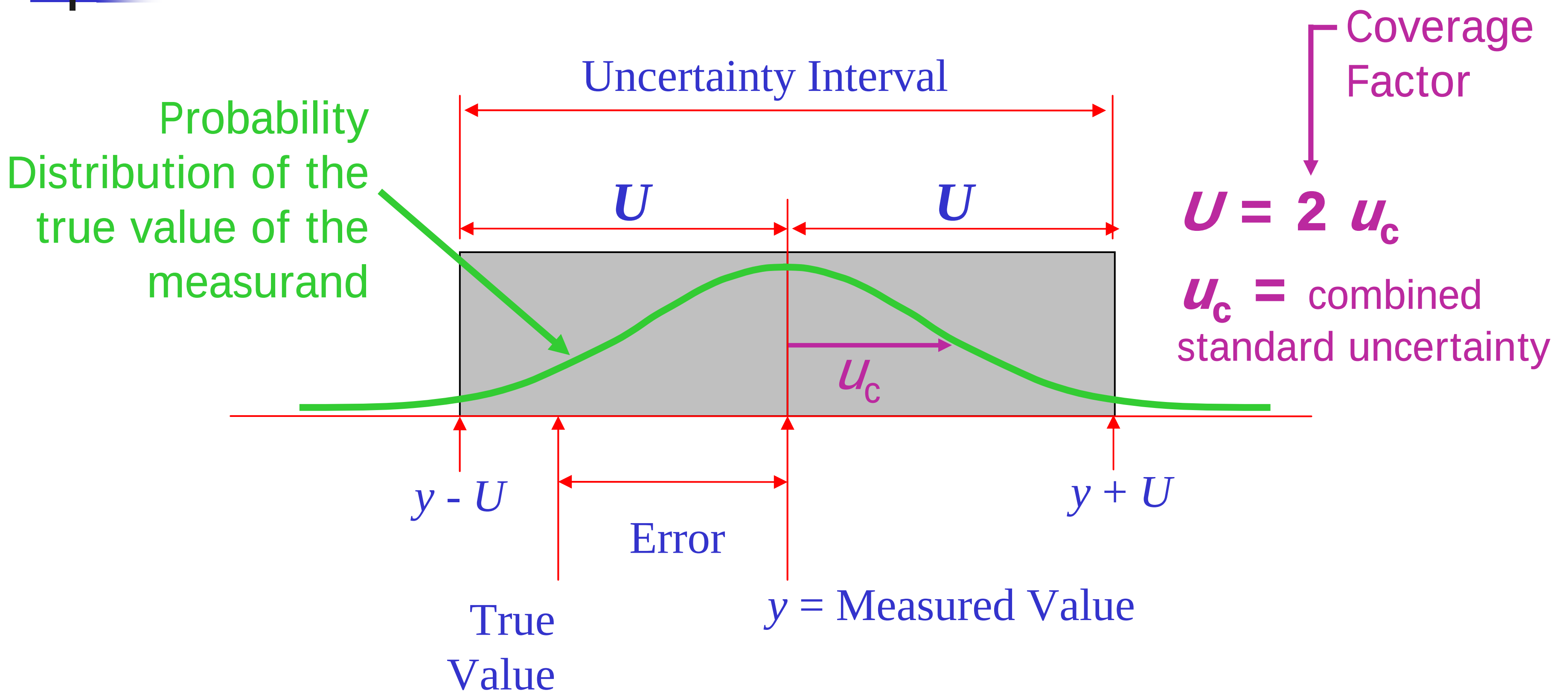


ASME B89.7.5: Dimensional Measurement Traceability



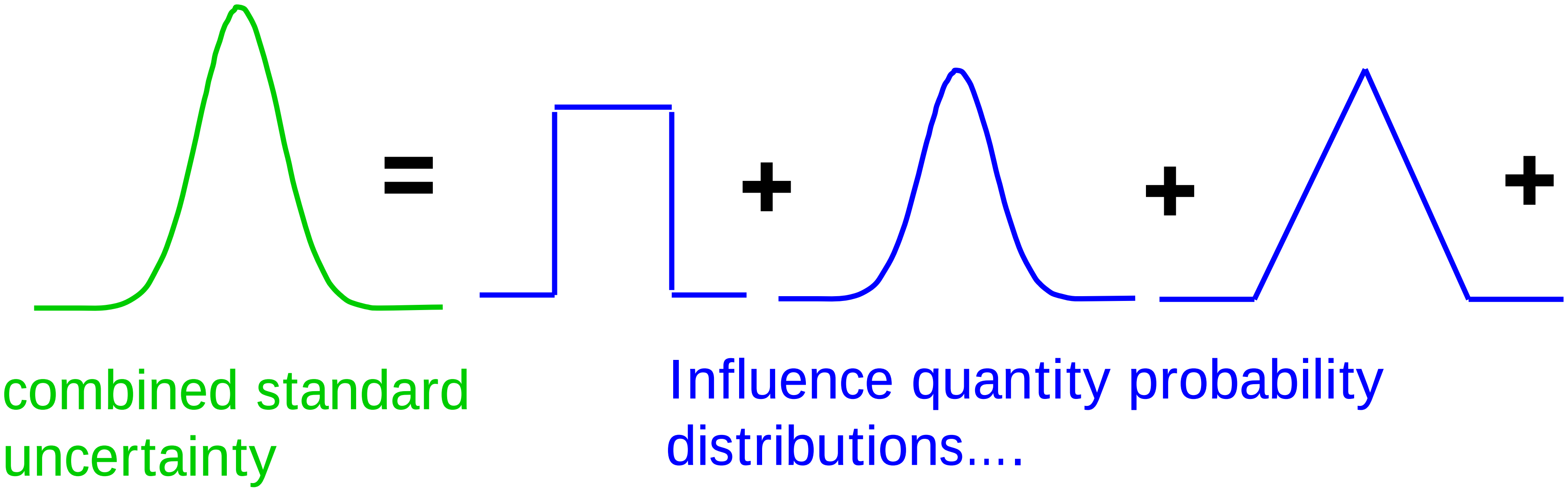
- Provides guidance on dimensional measurement traceability especially for industrial measurements
- Provides one specific interpretation of traceability that providers and users can agree upon
- Provides worked examples of traceability requirements

Overview of Uncertainty Evaluation



Overview of Uncertainty Evaluation

Key Point:
Combine Probability Distributions



Overview of Uncertainty Evaluation

There are several methods to combine distributions...

- Mathematical Analysis :

$$u_c^2 = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) \quad + \quad 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} r(x_i, x_j) u(x_i) u(x_j)$$

- Experimental Observations:

  "Super GR&R"

- Monte Carlo Calculations

  Computer Simulation ... Up Next!

Review of Basic Terminology

“A direct uncertainty of –5 μm resulted from a type B error associated with the systematic uncertainty of the resolution of the true value to yield a 95% confidence interval.”

“An expanded (k=2) uncertainty of *5 μm* included an input quantity from a type B uncertainty evaluation associated with the systematic effect of the resolution of the measuring instrument to yielding an uncertainty interval having a 95% level of confidence.”